



A PEDAGOGICAL FRAMEWORK FOR DEVELOPING ELEMENTARY SCHOOL STUDENTS' SPATIAL THINKING THROUGH GEOMETRIC TRANSFORMATIONS

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Abstract

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Spatial thinking is a fundamental cognitive ability that plays a crucial role in understanding and reasoning about geometric concepts, particularly geometric transformations. This study aimed to examine the effectiveness of a pedagogical framework designed to develop elementary school students' spatial thinking through geometry transformation learning. A quasi-experimental method with a one-group pretest-posttest design was employed involving 28 upper-grade elementary school students. The pedagogical framework integrated contextual problems, visual and concrete exploration, guided spatial reasoning, representational transformation, and reflection within learning activities on translation, reflection, rotation, and dilation. Students' spatial thinking abilities were assessed across four aspects: spatial visualization, mental rotation, spatial orientation, and geometric transformation reasoning. Data were analyzed using descriptive statistics and a Wilcoxon Signed Rank Test. The results indicated a significant improvement in students' spatial thinking abilities, with a notable increase in post-test scores compared to pre-test scores. The Wilcoxon Signed Rank Test revealed a statistically significant difference ($p = 0.000$), indicating a real and significant change in students' spatial thinking abilities. The standardized test statistic was 4.625, with a test statistic value of 406.000 and a standard error of 43.890, which demonstrates a strong practical impact of the intervention. These findings demonstrate that the proposed pedagogical framework is highly effective in enhancing elementary school students' spatial thinking and offers a meaningful instructional approach for geometry learning at the primary education level.

Keywords: Elementary School; Geometric Transformation; Pedagogical Framework; Spatial Thinking

Abstrak

Berpikir spasial merupakan kemampuan kognitif fundamental yang berperan penting dalam memahami dan menalar konsep-konsep geometri, khususnya transformasi geometri. Penelitian ini bertujuan untuk mengkaji efektivitas kerangka pedagogis yang dirancang untuk mengembangkan kemampuan berpikir spasial siswa sekolah dasar melalui pembelajaran transformasi geometri. Penelitian ini menggunakan metode kuasi-eksperimen dengan desain one-group pretest-posttest yang melibatkan 28 siswa sekolah dasar kelas atas. Kerangka pedagogis yang diterapkan mengintegrasikan permasalahan kontekstual, eksplorasi visual dan konkret, penalaran spasial terbimbing, transformasi representasi, serta refleksi dalam kegiatan pembelajaran translasi, refleksi, rotasi, dan dilatasi. Kemampuan berpikir spasial siswa dinilai berdasarkan empat aspek, yaitu visualisasi spasial, rotasi mental, orientasi spasial, dan penalaran transformasi geometri. Data dianalisis menggunakan statistik deskriptif dan uji Wilcoxon Signed Rank. Hasil penelitian menunjukkan adanya peningkatan yang signifikan dalam kemampuan berpikir spasial siswa, yang ditunjukkan oleh kenaikan skor posttest dibandingkan dengan skor pretest. Hasil uji Wilcoxon Signed Rank menunjukkan perbedaan yang signifikan secara statistik ($p = 0,000$), yang mengindikasikan adanya perubahan signifikan dalam kemampuan berpikir spasial siswa. Nilai statistik uji terstandarisasi sebesar 4,625, dengan nilai statistik uji 406,000 dan standard error sebesar 43,890, yang menunjukkan adanya dampak praktis yang kuat dari intervensi pembelajaran. Temuan ini menunjukkan bahwa kerangka pedagogis yang diusulkan efektif dalam meningkatkan kemampuan berpikir spasial siswa sekolah dasar serta menawarkan pendekatan pembelajaran yang bermakna untuk pembelajaran geometri pada jenjang pendidikan dasar.

Kata Kunci: Sekolah Dasar, Transformasi Geometri, Kerangka Pedagogis, Berpikir Spasial.



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1. Introduction

Spatial thinking is one of the fundamental cognitive abilities that plays an important role in understanding, representing, and reasoning the relationships between objects in space (Ishikawa, 2021; Newcombe & Shipley, 2015). In general, spatial thinking is understood as a mental process that involves the ability to represent, analyze (Liu et al., 2023), as well as drawing inferences from the spatial relationships between objects or their parts (McNeal et al., 2021). This ability is not only limited to shape recognition, but also includes mental manipulation of objects, such as performing three-dimensional rotations, cross-sectional cutting, as well as various forms of visual transformation in the mind (Bairaktarova et al., 2015; Reyes et al., 2016).

Further, spatial thinking is closely related to the ability to visualize and recognize patterns. Individuals with good spatial thinking skills are able to visualize three-dimensional objects, identify patterns, and explain the relationships between objects in a given context (Lashari et al., 2024). Spatial thinking skills have proven to be relevant in a variety of fields, such as engineering, architecture, graphic design, and computer science, which demand a deep understanding of form, structure, and spatial relations. Spatial thinking skills are key in understanding abstract geometric concepts, such as geometric transformations and relationships between spatial structures, because students need to imagine how an object changes or interacts in space (Agustina et al., 2025).

Spatial thinking also helps students connect three-dimensional representations into two-dimensional shapes, which are fundamental skills in school geometry. This process involves not only visual imagination, but also spatial reasoning to accurately interpret mathematical images, diagrams, and symbols (Fiantika et al., 2018). This suggests that spatial problem-solving is a complex cognitive process and requires appropriate learning support (Stieff et al., 2010). A number of studies have also shown that spatial thinking is multidimensional and can vary between individuals. Some students show excellence in three-dimensional visualization, while others are better able to explain solutions verbally and use two-dimensional representations (Nite et al., 2023; Rahmawati et al., 2021). In addition, the results of other studies confirm that spatial thinking is not a single ability, but a set of skills that can be developed through diverse learning

experiences, especially in the context of geometry learning (Choo et al., 2021; Zhu et al., 2023).

The urgency of developing spatial thinking is getting stronger at the basic education level. Elementary school is a crucial phase in the formation of a student's cognitive foundation, including the ability to reason logically and solve problems. Spatial thinking developed from an early age has been shown to have a strong relationship with learning success, as it supports advanced concept understanding and reasoning at the next level of education (Burte et al., 2017; Gagnier & Fisher, 2020). Therefore, strengthening spatial thinking in elementary schools is a long-term investment for students' academic development. Various learning interventions have shown effectiveness in improving the spatial thinking skills of elementary school students. Training programs such as Think3d! which involves motion-based activities and concrete manipulation has been proven to be able to improve spatial abilities while having a positive impact on students' mathematical performance (Holly A. & Hutton, 2013; Lowrie et al., 2021a; Pellas, 2025). However, the success of the development of spatial thinking is highly dependent on the role of teachers. Research shows that there are still many primary school teachers who have limited understanding and confidence in integrating spatial thinking into learning (Gagnier & Fisher, 2020). This condition confirms the need for a clear and structured pedagogical framework. Geometry transformation is one of the core materials in geometry learning that has a significant contribution to the development of students' spatial thinking (Dwirahayu et al., 2025; Turgut, 2022). Through translational, reflective, rotational, and dilation activities, students are trained to mentally imagine changes in the position, shape, and size of objects, thereby strengthening their spatial visualization and reasoning skills. However, various studies show that elementary school students still face conceptual difficulties in understanding geometric transformations, both in identifying the type of transformation and in predicting the outcome of a transformation (Andrés et al., 2023; Sunariah & Mulyana, 2020; Tarng et al., 2024). These difficulties are exacerbated by learning that still emphasizes procedural aspects without providing a deep conceptual understanding. As a result, students tend to encounter obstacles when it comes to applying the concept of geometric transformation in different contexts. To address this problem, studies recommend the use of dynamic technologies, exploratory learning approaches, and the development of structured learning units to help students build a meaningful understanding of geometric transformations (Avcu & Çetinkaya, 2021; Bímová et al.,

2020; Panorkou & Maloney, 2015). Geometry learning in elementary school is still dominated by procedural and symbolic approaches that emphasize formulas and mechanical exercises, so students focus more on the end result than the spatial thinking process. The lack of visual, manipulative, and exploratory activities causes students to have difficulties in visualizing objects, performing mental rotations, and understanding the spatial relationships between buildings, which has an impact on the low conceptual understanding of geometric transformation.

Previous research has shown that visualization-based learning and manipulative activities are effective in improving spatial thinking skills and geometry understanding. However, the study of spatial thinking and geometric transformation is generally still separate and has not been integrated into a systematic pedagogical framework and empirically tested at the elementary school level. Therefore, this study proposes a pedagogical framework that integrates geometric transformation learning with the development of spatial thinking and is tested through experimental design

2. Research Methods

This study uses a quasi-experimental method with a one-group pretest–posttest design to test the effectiveness of pedagogical framework in developing the spatial thinking skills of elementary school students through geometry transformation learning. The quasi-experimental design allowed researchers to compare students' spatial thinking skills before and after learning interventions in the same classroom context (Creswell, 2014; Shadish, 2002).

Research Subjects and Settings

The subjects of the study were high grade elementary school students at SD Pekayon 03 Cipayung. The selection of subjects was carried out using purposive sampling, taking into account that students are studying geometry transformation material in accordance with the elementary school mathematics curriculum. The research was carried out during regular mathematics learning at SD Pekayon 03 Cipayung, so that the intervention took place in a natural and contextual learning environment (Anderson & Shattuck, 2012).

Research Instruments

Students' spatial thinking abilities were measured using spatial thinking tests that included aspects of spatial visualization, mental rotation, spatial orientation, and geometric transformation reasoning. The indicators measured include the ability to identify and visualize changes in the shape and position of the building due to translation, reflection, rotation, and dilation, imagine the mental rotation of a building, and determine the spatial relationship between the position and orientation between buildings. The instruments (Table 1) are arranged based on spatial thinking constructs commonly used in mathematics education research (Hegarty & Waller, 2005). The validity of the instrument is obtained through expert judgment, while the reliability of the instrument is tested before it is used in the study.

Data Collection Procedure

Data collection began with the provision of a pretest of spatial thinking skills to students before the implementation of the pedagogical framework. After the entire learning series is completed, students are given a posttest using the same instrument. This procedure aims to identify changes in students' spatial thinking abilities as a result of learning interventions (Fraenkel & Wallen, 2012).

Data Analysis Techniques

Data were analyzed using descriptive statistics to obtain the average score and standard deviation of pretest and posttest scores. Furthermore, a paired sample t-test was carried out to determine the significant differences between students' spatial thinking skills before and after the learning intervention. This analysis technique is appropriate for paired data coming from the same subject at two measurement times (Field, 2018).

Table 1. Grid of the Student Spatial Thinking Ability Test Instrument

Aspects of Spatial thinking	Indicator	Question Number	Number of Questions
Spatial Visualization	Identify geometric shapes and positions before and after transformations	1, 2	2
	Determining the visual image of the translation and reflection	3, 4	2

Mental Rotation	Imagine a change in the position of the building due to rotation at a certain angle	5, 6	2
	Determining the orientation of the wake up after the rotation	7	1
	Determining the position relationship between buildings in a space or coordinate field	8, 9	2
Spatial Orientation	Determining the location and direction of the dilated walking shadow	10	1
	Determine the type of transformation that occurs based on changes in the position and shape of the building	11, 12	2
	Explain the invariant nature of the wake after transformation (shape, size, orientation)	13	1
Geometry Transform Reasoning			
Total		13 Questions	

3. Results and Discussion

Pedagogical Framework

This pedagogical framework (Figure 1) is designed to develop the spatial thinking skills of elementary school students through meaningful, visual, and studentcentered geometric transformation learning. This framework integrates the concepts of geometric transformation (translation, reflection, rotation, and dilation) with the main components of spatial thinking, namely spatial visualization, mental rotation, and spatial orientation. Learning is structured in stages so that students build conceptual understanding as well as spatial reasoning skills through active and contextual learning experiences.

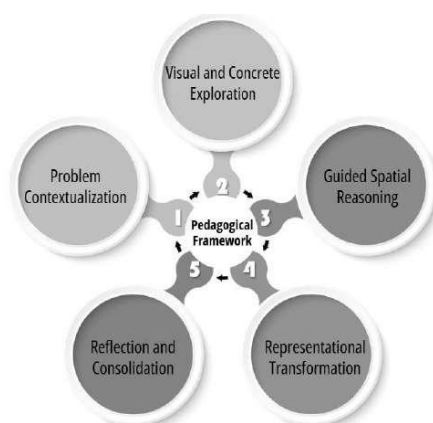


Figure 1. Pedagogical Framework

The problem contextualization stage aims to activate students' initial knowledge through contextual problems related to changes in the position and shape of objects in the surrounding environment. At this stage, students are invited to observe the phenomenon of transformation intuitively so that cognitive needs are formed to understand the concepts of geometry learned. The visual and concrete exploration stage emphasizes the use of concrete visual and manipulative media, such as drawings, squared paper, flat builds, or simple props. Through the activities of moving, rotating, and reflecting objects directly, students develop spatial visualization skills and intuitively understand the characteristics of each type of geometric transformation.

The guided spatial reasoning stage is focused on the development of spatial reasoning through triggering questions, guided discussions, and high-level thinking activities. The teacher acts as a facilitator who helps students explain changes in position, orientation, and size of the body, as well as gradually practice mental rotation and spatial orientation skills. The representational transformation stage encourages students to connect concrete experiences with symbolic and graphic representations. At this stage, students represent the results of the transformation in the form of simple images, coordinates, or notation, thus strengthening conceptual understanding and the relationship between visual and mathematical representations.

The last stage, reflection and consolidation, aims to strengthen and integrate students' understanding. Students reflect on the thought process that has been carried out, compare different types of transformations, and conclude the main concepts learned. This reflection helps students build metacognitive awareness of the spatial thinking strategies used. Overall, this pedagogical framework places geometric transformation not only as a procedural material, but as a strategic means to systematically train students' spatial thinking skills. The integration of visual, manipulative, and reflective activities in this framework provides a more meaningful learning experience and supports the holistic development of elementary school students' cognitive abilities.

The pedagogical framework developed in this study is implemented in mathematics learning in elementary schools, especially in grade V students. At this stage of cognitive development, students also have the readiness to develop visualization, mental rotation, and spatial orientation skills in a more structured manner. Table 2 are the results of descriptive statistics to illustrate the differences in

students' spatial thinking skills before and after the application of geometry transformation learning based on the pedagogical framework. The data displayed included the number of subjects, mean scores, median, standard deviation, and error standards as the basis for the comparative analysis of the pretest and posttest.

Table 2. Descriptive Data Analysis Results

	N	Mean	Median	SD	SE
Pre Test	28	54.5	54.5	4.23	0.799
Post Test	28	76.5	76.0	4.48	0.846

The results of descriptive statistics show an increase in students' spatial thinking skills after the application of geometry transformation learning based on *the pedagogical framework*. The average score of the students' pretest ($N = 28$) was 54.5 with a standard deviation of 4.23, while the average score of the posttest increased to 76.5 with a standard deviation of 4.48. The average difference of 22 points indicates a considerable and relatively even improvement in spatial thinking skills in most students. Descriptively, these findings show that applied learning has the potential to be effective in improving the spatial thinking skills of elementary school students. Furthermore, Table 3 presents the results of the normality test of research data.

Table 3. Result of Tests of Normality

			statistic	p
Pre Test	Post Test	Shapiro-Wilk	0.728	$P < 0,001$
		Kolmogorv-Smirnov	0.379	$P < 0,001$
		Anderson-Darling	3.93	$P < 0,001$

Based on the results of the normality tests conducted using Shapiro-Wilk, Kolmogorov-Smirnov, and Anderson-Darling, the p-values were all less than 0.05, i.e., $p < 0.001$. This indicates that the pre-test and post-test data were not normally distributed. Thus, the normality assumption for using parametric analysis such as the paired-samples t-test was not met. Therefore, to analyze the differences between the pre-test and post-test, the next step is to use a non-parametric analysis, namely the Wilcoxon Signed-Rank test, which is more appropriate for data that is not normally distributed. Table 4 shows the result for the Signed-Rank Test.

Table 4. Results of the Hohypothesis Test Analysis

	Null Hypothesis	Test	Sig.	Decision
1	The median of differences between Pre_Test and Post_Test equals 0.	Related-Samples Wilcoxon Signed Rank Test	.000	Reject the null hypothesis.

Based on the results of the Wilcoxon Signed Rank Test, a significance value (p-value) of 0.000 was obtained, which is smaller than 0.05 (the commonly used significance limit). This indicates that the null hypothesis stating that there is no significant difference between the pre-test and post-test is rejected. In other words, there is a significant difference between the pre-test and post-test scores.

Based on the displayed Wilcoxon Signed Rank Test results (Table 5), the Test Statistic value was 406,000, with a Standard Error of 43,890, and a Standardized Test Statistic of 4,625. The Asymptotic Sig. (2-sided test) value obtained was 0.000, which is much smaller than the significance limit of 0.05. This indicates that the difference between the pre-test and post-test scores is statistically significant. Thus, we can reject the null hypothesis and conclude that there is a significant difference between the pretest and post-test. This indicates that the intervention carried out has had a real impact on improving the measured abilities, which supports the hypothesis that the changes that occurred were not coincidental, but rather the result of the applied learning application.

Table 5. Related-Samples Wilcoxon Signed Rank Test Summary

Related-Samples Wilcoxon Signed Rank Test Summary	
Total N	28
Test Statistic	406.000
Standard Error	43.890
Standardized Test Statistic	4.625
Asymptotic Sig. (2-sided test)	.000

These results indicate an increase in spatial thinking skills after the implementation of geometric transformation learning based on a pedagogical framework. This increase reflects the effectiveness of this learning approach in developing students' spatial thinking skills, which are very important in understanding geometric concepts at the elementary level.

Aspects of Spatial Visualization

In the aspect of spatial visualization, students show an improvement in their ability to identify the shape and position of geometric shapes before and after transformation, as well as determine the visual image of the building as a result of translation and reflection. After learning, students are not only able to recognize changes in waking position, but also understand the visual relationship between early waking and their shadow.

Geometry-based transformation learning *pedagogical framework* provide opportunities for students to observe, manipulate, and visually represent waking through a variety of contextual and representational activities. This is in line with spatial visualization theory which states that spatial abilities develop when students are trained to construct and transform mental representations of objects (Clements, 2000; Clements et al., 2003; Lohman, 1993). Other research results also support these findings which show that geometry learning based on visual representation and transformation activities can significantly improve students' spatial visualization skills (Lowrie et al., 2021b; Medina Herrera et al., 2024).

Aspects of Mental Rotation

In the aspect of mental rotation, the increase in ability can be seen in the indicator to imagine a change in the waking position due to rotation at a certain angle and determine the orientation of the waking after experiencing rotation. Students become more skilled at predicting rotation outcomes without having to rely on concrete images all the time. Mental rotation ability is one of the core components of spatial thinking which is related to the ability to manipulate objects mentally (Shepard & Metzler, 1971). Through learning that emphasizes the exploration of geometric transformations gradually, students are trained to imagine the rotation and understand the orientation changes that occur.

These findings are in line with this study which shows that rotational exercises through structured geometry learning can improve the mental rotation abilities of elementary school students (Diago et al., 2022; Hawes et al., 2015). Therefore, a significant improvement in this aspect suggests that applied learning successfully trains intermediate-level spatial thinking skills that require complex mental manipulation.

Aspects of Spatial Orientation

In the aspect of spatial orientation, students experience an increase in their ability to determine the position relationship between buildings in space or coordinate planes, as well as determine the location and direction of the shadow of the dilated building. Students become better able to understand the relationships of position, direction, and scale in the context of space and field. Geometry transformation learning provides a learning experience that emphasizes the spatial relationships between objects, so that students not only understand building separately, but also in coordinate systems and spatial relations. The results of this study reinforce this finding that geometry learning based on transformation and coordinate activities is effective in improving students' spatial orientation (González-Campos et al., 2022; Turgut, 2022).

Aspects of Geometry Transformation Reasoning

In the aspect of geometry transformation reasoning, the improvement in students' abilities can be seen in indicators determining the type of transformation that occurs based on changes in the position and shape of the building, as well as explaining the invariant nature of the building after the transformation (shape, size, and orientation). Students are not only able to identify transformations, but also understand the concept of invariance in geometric transformations. This aspect shows the development of a high level of spatial thinking skills that involve reasoning and generalization. According to van Hiele's theory, the ability to understand the nature of invariants and the relationships between transformations suggests that students have moved towards a more formal level of geometric thinking (Burger & Shaughnessy, 1986). The results of this study are consistent with the findings that transformation-based geometry learning is able to improve spatial reasoning skills and deep understanding of geometry concepts (Awaluddin et al., 2023; Zhao et al., 2024).

4. Conclusion and Suggestions

Based on the results of the Wilcoxon Signed Rank Test, a significance value (pvalue) of 0.000 was obtained, which is smaller than the commonly used significance limit of 0.05. This indicates that the null hypothesis, which states that there is no significant difference between the pre-test and post-test, is rejected. In other words, there is a significant difference between the pre-test and post-test scores. The Wilcoxon Signed Rank Test results showed a Test Statistic value of 406.000, with a Standard Error of 43.890, and a Standardized Test Statistic of 4.625. The Asymptotic Sig. (2-sided test) value obtained was 0.000, which is much smaller than the significance limit of 0.05, confirming that the difference between the pre-test and posttest scores is statistically significant. Thus, we can reject the null hypothesis and conclude that there is a significant difference between the pre-test and post-test. This result indicates that the intervention carried out has had a real impact on improving the measured abilities. The findings support the hypothesis that the observed changes were not coincidental, but rather the result of the applied learning method. The significant improvement observed in the spatial thinking abilities of the students suggests that the learning intervention has been effective in enhancing various aspects of spatial thinking, including visualization, mental rotation, spatial orientation, and geometric transformation reasoning. This demonstrates that the learning method applied contributed to the overall development of the students' spatial thinking skills, from basic visual skills to high-level geometric reasoning.

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